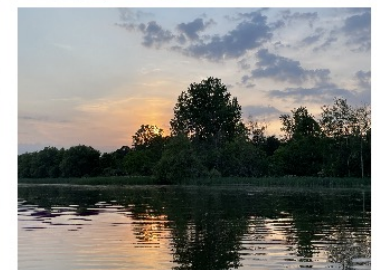
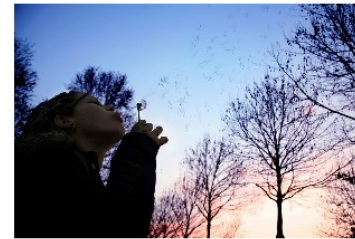


Problems with Purpose

Volume 1

October 23, 2023



Source: Top left photo: https://commons.wikimedia.org/wiki/File:Woman_blowing_dandelion_seeds_in_Paris.jpg

Source: Top right photo: [https://commons.wikimedia.org/wiki/File:Sierra227s_birthday_campfire_\(16209461406\).jpg](https://commons.wikimedia.org/wiki/File:Sierra227s_birthday_campfire_(16209461406).jpg)

Source: Bottom left photo: <https://commons.wikimedia.org/wiki/File:A08-16-2093.jpg>

Source: Bottom right photo by Judith Koeller

Problems with Purpose

Free at <https://cemc.uwaterloo.ca/resources/problems-with-purpose.php>

Territorial Acknowledgement

I live and work on the traditional territory of the Neutral, Anishinaabeg, and Haudenosaunee peoples. UWaterloo's main campus is situated on the Haldimand Tract, the land granted to the Six Nations that includes six miles on each side of the Grand River.



Six Nations Lands & Resources,

<https://www.sixnations.ca/LandsResources/ClaimSummaries.htm>



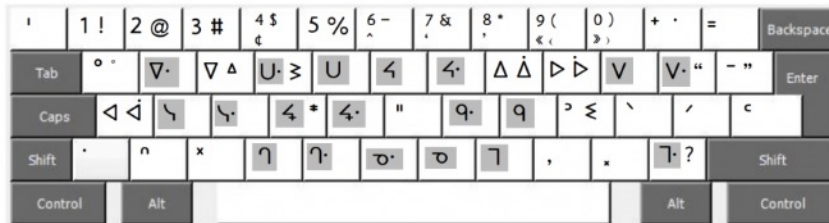
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Territorial Acknowledgement (more personally)

- One theme I'll mention is about language.
- Language is a vital part of culture. Colonialism actively suppressed Indigenous languages.
- Math and computer science can have an impact, both positive and negative, on Indigenous languages.

Cree - Western Keyboard Layout



Copyright 2010 Chris Harvey

Māori are trying to save their language from Big Tech

Te Hiku Media gathered huge swathes of Māori language data. Corporates are now trying to get the rights to it



GETTY IMAGES / TIM GRAHAM / CONTRIBUTOR



Centre for Education in Mathematics and Computing (CEMC)

Inspiring students to reach further with mathematics and computer science

- Housed within the Faculty of Mathematics at the University of Waterloo, 40 faculty and staff, and hundreds of volunteers
- Scope: grades 3 – 12
 - 20,000 students benefit each year from the **free CEMC Visits Schools program** at more than 300 schools in over 15 countries (in-person or online)
 - **Free online courseware** receives 10 million pageviews annually
 - 300 teachers are currently taking our **on-line Master of Mathematics for Teacher's** program, with scholarships available to folks who are Indigenous or have strong connections to Indigenous communities
 - 1.5 million learners and teachers benefit from our **Problem of the Week**
 - **Contests**: 265,000+ students in 80+ countries annually





<https://cemc.uwaterloo.ca>

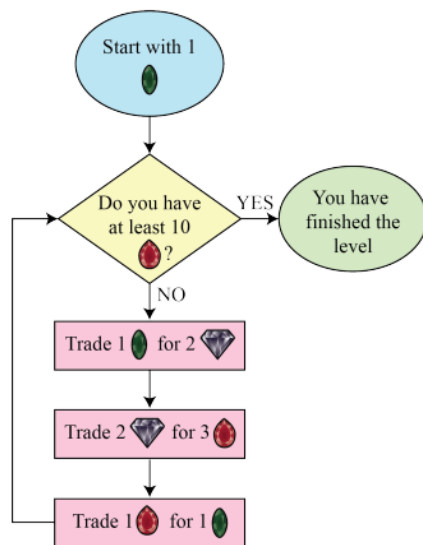
Centre for Education in Mathematics and Computing (CEMC)

Problem of the Week


Problem A

Gathering Treasure

Genevieve is making a video game where players need to trade gems in order to get to the next level. The gems in the game are emeralds , diamonds , and rubies . In the first level, players make three trades of their gems, as shown in the diagram, until they have at least 10 rubies .



- How many of each gem will a player have when they finish the first level?
- How many trades in total will a player have made when they finish this level?

10.  Suppose that s and t are real numbers with $0 < s \leq 1$ and $0 < t \leq 1$. Points $A(-1, 0)$, $B(0, 4)$ and $C(1, 0)$ form $\triangle ABC$. Points $S(s, 0)$ and $T(-t, 0)$ lie on AC . Point P lies on AB and point Q lies on BC , with neither P nor Q at a vertex of $\triangle ABC$. Line segments SP and TQ intersect at X and partition $\triangle ABC$ into four regions. For some such pairs (s, t) of real numbers and points P and Q , the line segments SP and TQ in fact partition $\triangle ABC$ into four regions of equal area. We call such a pair (s, t) a *balancing pair*.

(a) Suppose that (s, t) is a balancing pair with $s = 1$ and that line segments SP and TQ partition $\triangle ABC$ into four regions of equal area. Determine the coordinates of P .

(b) Prove that there exist real numbers d , e , f , and g for which all balancing pairs (s, t) satisfy an equation of the form

$$s^2 + t^2 = dst + es + ft + g$$

and determine the values of d , e , f , and g .

(c) Determine an infinite family of distinct pairs of rational numbers (s, t) with $0 < s \leq t \leq 1$ that satisfy the equation in (b).

Check Your Understanding 1

Determine the missing values in the table. The data is quadratic.



About Judith

- Professor in the CEMC since 2003
- Run math/CS outreach sessions in hundreds of schools
 - Across Turtle Island and internationally
 - Indigenous communities
 - STEM day camps, Girl Guides of Canada



Why “Problems with Purpose”?

- Obstacle to student engagement in math: “would we use this in the real world?”
- CEMC listened to teachers who are Indigenous, and teachers who teach in Indigenous communities about how CEMC could support the calls of the TRC.
- Recurring themes
 - importance of exploring math through **hands-on** activities
 - **concrete** connections to the world around us
 - experiencing **math as a tool that helps** communities and the world
- **Problems With Purpose** is a collection of mathematical problems to be used in grades 4 to 12.
- Each problem is meant to highlight the connection between mathematics and our collective responsibility to present and future generations.
- Free at <https://cemc.uwaterloo.ca/resources/problems-with-purpose.php>



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Math Keywords

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Commissioned the Photographer **Bangishimo**

Bangishimo (They/Them) is an IndigiQueer Anishinaabe, originally from Couchiching First Nation located on Treaty #3 territory. Bangishimo's focus is creating space for communities to come together, allowing for Black, Indigenous and racialized voices to be heard. They are a co-founder of **O:se Kenhionhata:tie**, also known as Land Back Camp. Their advocacy and photography has allowed them to visit over sixteen countries; taking photos and sharing the stories of those they meet along the way. Bangishimo's work has been featured in numerous publications and displayed throughout Waterloo Region. Bangishimo won the Briarpatch Writing in the Margins Contest - Photography Category and was voted Best Photographer 2021 in the Community Edition.

Source: <https://www.bangishimo.ca/>

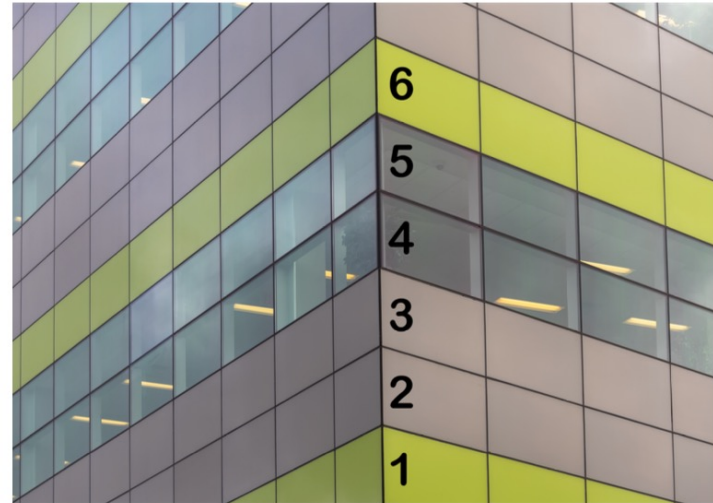


<https://www.bangishimo.ca>

Bird-Safe Glass (grades 4 – 8)

The building shown has some rows of panels in clear glass (like the fourth and fifth rows), some rows in green (like the first and sixth rows) and some rows in gray (like the second and third rows). The colours of the rows form a pattern that repeats after every five rows.

Suppose that the building needs to have all of its clear glass panels updated to bird-safe glass. Since birds often cannot see clear glass, bird-safe glass is made with a pattern on it that birds can see.



Source: Photo by Bangishimo

1. The building has 26 rows of panels in total. How many rows of clear glass does it have?
2. On each side of the building, each row contains 12 panels. How many panels in the whole building have clear glass?
3. The cost to replace one clear glass panel with bird-safe glass is \$150. What is the total cost to replace all the building's clear glass panels with bird-safe glass?

Here are some other [ways you can protect birds so they don't fly into glass.](#)

Collaborated with Dr. Edward Doolittle, First Nations University (originally from Six Nations)



Name: Edward Doolittle, *Mohawk*

Discipline: Theoretical Mathematics

University/Year: University of Toronto, 1997

Current Position: Associate Professor of Mathematics, First Nations University of Canada

Research Interests:

- Partial Differential Equations
- Probability
- Indigenous Mathematics
- Mathematics Education

Bio:

Dr. Edward Doolittle is Kanyenkehake (Flint Nation = Mohawk) from Six Nations in southern Ontario. He earned his PhD in pure mathematics (partial differential equations) from the University of Toronto in 1997. In 2000-2001 he was a member of Onkwewenna Kentsyohkwa (Our Language Group) studying the Mohawk language full time in immersion in his home community. In 2001, he joined the faculty of First Nations University of Canada, a federated college of the University of Regina, where he is now Associate Professor of Mathematics.

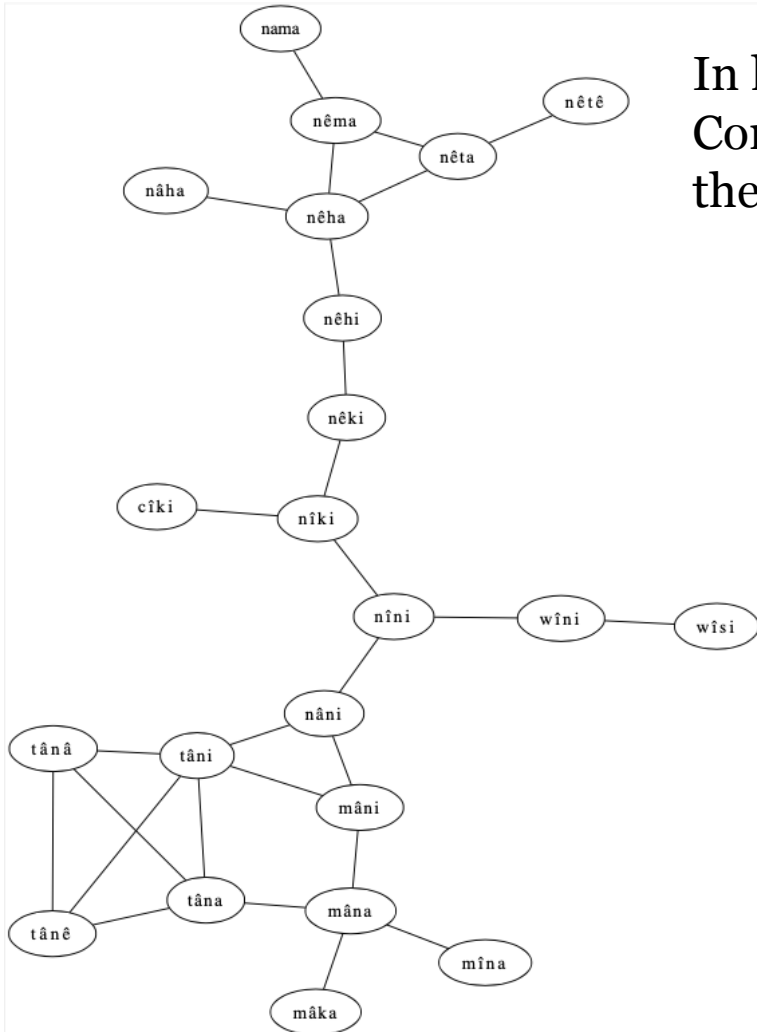
<https://indigenoumathematicians.org/edward-doolittle/>



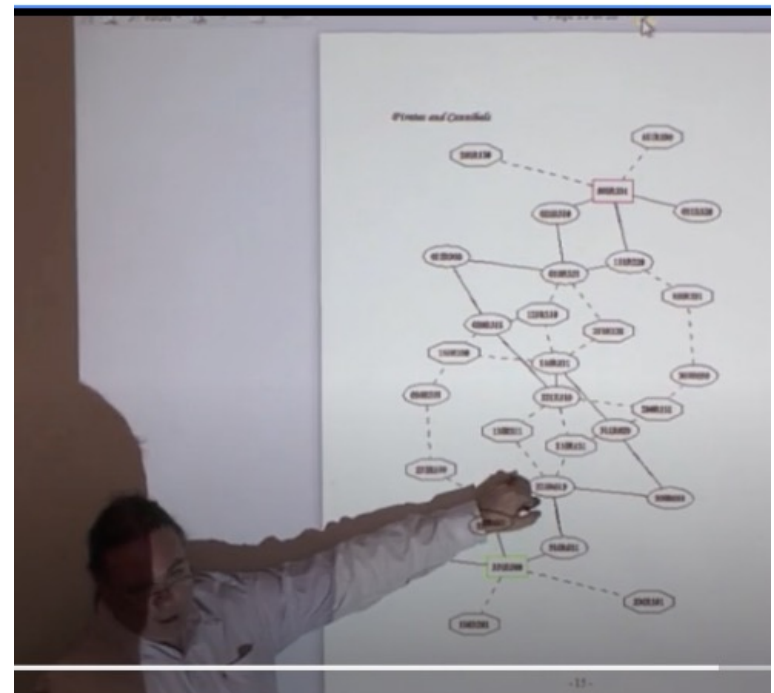
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Graph Theory to Analyze the Cree Language(s)



In his 2012 talk “Graph Theory in an Indigenous Context”, Dr. Doolittle describes his use of graph theory to analyze patterns in the Cree language.



Left image by Dr. E. Doolittle

Right image from <https://www.birs.ca/events/2012/5-day-workshops/12w5076/videos/watch/201211210914-Doolittle.html>.



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Dr. Doolittle Creates Word Games to foster the Cree Language

Match the Cree word with the appropriate English word or phrase.
Then find each of the Cree and English words in the square below.

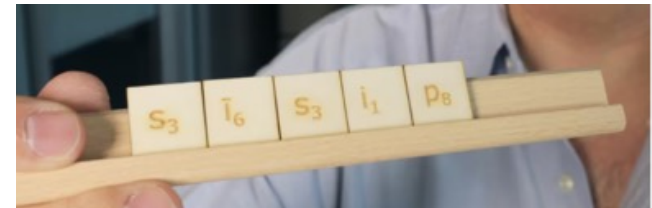
p i s i w ē n ā w k i t s o k h i m s e
i o s ā w i s i p i s ē t i k s a k i s
s n ō s ē - n i s k a g o s l i n g h n
o n n ā p ē - n i s k a w ē h w ē w k i
k k ā h k ā k i s i p i s i n i y i i i
s s i h k i h p i s i s i p i s i s h k
i s i s i k s i n - i c s i p a a m p c
m a t - m k f e m a l e g o o s e i u
a t s k i h c i - n i s k a p i s i s d
c g n i l k c u d n e h r e t a w ē k
o a s c a n a d a g o o s e r o s e
r l a u g h i n g g o o s e i e o o i
m n r e d - h e a d e d d u c k d n o
o k c u d n w o r b l r w d r ā t
r a k c u d k c a l b a a k s e d p c
a k c u d e g r a l s w m l b d u ē o
n s g s s n o w g o o s e e l n c s n
t i m i s i - s i s i p h s g f a k i -
d n e h r e t a w d u c k l i n g m p ē
g o o s e k a r d p d w s n i e r u

amiskosip	black duck
apisci-niskis	brown duck
iyinisip	canada goose
kaskitēsip	cormorant
kihci-niska	drake (male duck)
kāhkākīsip	duck
mihkostikwānēwisip	duckling
misī-sīsīp	female duck
niska	female goose
niskisis	gander (male goose)
nāpē-niska	goose
nāpēsīp	gosling
nōsē-niska	large duck
nōsēsīp	laughing goose
osāwisip	mallard
sihkihp	red-headed duck
sihkihpīsīp	snow goose
sīsīp	waterhen
sīsīpīsīp	waterhen duckling
wēhwē	wood duck

Academic at First Nations University turning Scrabble into Cree language game

Associate professor of mathematics keeps the Scrabble board, but makes new tiles for 'Screbble'

CBC News · Posted: Sep 26, 2019 8:03 PM EDT | Last Updated: September 26, 2019



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Left image of Cree word search by Dr. E. Doolittle

Right images from CBC News, <https://www.cbc.ca/news/canada/saskatchewan/cree-scrabble-regina-1.5299049>

Problems with Purpose shines a light on his work

Cree Word Tree Puzzle (grades 4 - 6)

Cree is a group of Indigenous languages spoken by about 100,000 people. [Watch a 2-minute YouTube video](#) to hear five words in Cree.

Dr. Edward Doolittle studies word patterns in Cree, and uses mathematics and computer science [to create Cree word puzzles](#).



Source: <https://www.giftoflanguageandculture.ca/giftoflanguageandculture-ca-cree-language/> and <https://www.fnuniv.ca/academic/faculty/dr-edward-doolittle/>

p	a	s	p	i	n	ē
p	a	s	k	i	n	ē
p	ā	s	k	i	n	ē
t	ā	s	k	i	n	ē
t	ā	k	i	n	ē	

A **Word Tree** is a list of words where each word differs from the previous word by just one letter.

In the word tree to the left, notice that the Cree letter **a** is different from the letter **ā**. Similarly, the Cree letter **i** differs from **ī**, **o** differs from **ō** and so on.



1. In the word tree to the right, colour each letter that is different as you move down the list.

tāwinam
tawinam
kawinam
pawinam
pīwinam

2. The word tree to the right starts with the word “nakinam” and ends with the word “sikosam”. Fill in the word tree with the following five words, colouring each letter that is different as you move down the list.

sīkonam sakinam sikonam sīkinam sēkonam

nakinam

sikosam

3. The word tree to the right starts with “nikamow”, ends with “wākamon” and has six other words in between.

Create a new, shorter word tree that

- starts with “nikamow”,
- ends with “wākamon”,
- includes only words from the original word tree,
- and has only **four** other words in between.

nikamow
nakamow
sakamow
sākamow
wākamow
wāpamow
wāpamon
wākamon

nikamow

wākamon



Contributors

We are grateful to the many folks whose insights, ideas and skills contributed to this project:

Advisory:	Jean Becker	Kathleen Couch	Edward Doolittle	Nathan Rowbottom
	Mark Skanks	Rachel White		
CEMC:	Sandra Emms-Jones	Rob Gleeson	Brian Ingalls	Judith Koeller
	Ryan McGrath	Jen Nelson	Lata Punetha	Nicholas Rollick
	Ian VanderBurgh	Christine Vender		

What we are Learning/Challenges

- It takes time
 - To build meaningful relationships
 - To make good math problems
- Power of synergy
- Dangers of going off the rails
- Shining a light vs appropriation



Invitation to Collaborators for Problems with Purpose

- We hope to release two volumes per year.
- Recognizing the importance of reciprocity and avoiding appropriation...
If you have a theme that you'd like to see featured in a math question in Problems with Purpose, please contact me at jakoelle@uwaterloo.ca.
- We strive to work within the Faculty of Mathematics' Equity and Inclusive Communities Principles, including
 - Sharing collective responsibility
 - Respecting FNIM data sovereignty
 - Nurturing reciprocity with FNIM peoples



A Few More Samples

Tree Height

When managing forests, it can be helpful to know the height of a tree, but it isn't easy to measure tree height directly. The following method uses geometry/trigonometry to estimate tree height.



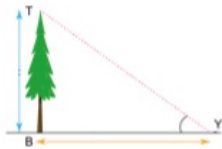
Step 1: Walk away from the tree, but at regular intervals bend down and look through your legs back up towards the tree.

Step 2: Continue doing this until you can just see the top of the tree.

Step 3: Measure the distance along the ground from your back to the tree.

Step 4: The distance you have measured is roughly equal to the tree's height.

1. In the diagram, the sides of the triangle are the ground (YB), the tree (BT), and your line of sight when looking at the top of the tree (YT).



Which two sides does the method assume are equal? In order for these two sides to be equal, what must $\angle BYT$ (your line of sight) equal?

2. If you use this method, what is one way to ensure that your line of sight is as needed?

Polar Bears

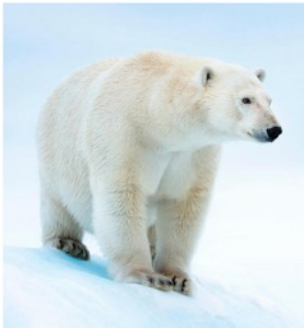


Image Source: <https://www.worldwildlife.org/species/polar-bear>

A team of biologists wish to estimate the size of a polar bear population within a specific region of the Arctic. On their first visit, they capture, tag and release 40 polar bears. On their second visit, they capture and release 60 polar bears, of which 15 already have tags.

- (a) What is the minimum polar bear population size possible? Can we determine a maximum?
- (b) Give a realistic estimate for the total polar bear population. What assumptions does your estimate rely upon?

Plant Sampling

Plant Sampling helps us to understand the interactions of plants, and to determine how fertile soil is.

A quadrat is a square frame that can be placed directly on top of vegetation.

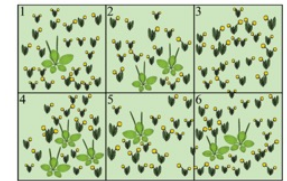


Image Source: [https://www.vetpages.uidaho.edu/veg_assess/Modules/Lessons/Module205\(Density\)/5_2-Plot-based_Techniques.htm](https://www.vetpages.uidaho.edu/veg_assess/Modules/Lessons/Module205(Density)/5_2-Plot-based_Techniques.htm)

A common perennial is *Plantago Major* (also called broadleaf plantain). It is not closely related to the fruit plantain, which is a kind of banana.



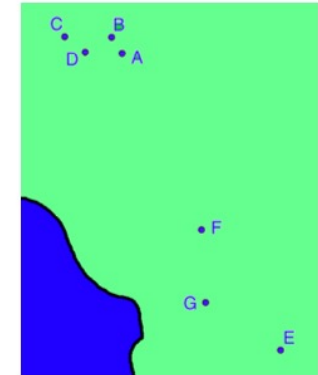
1. The diagram shows six quadrats, some of which contain *Plantago Major* plants. What proportion of quadrats have at least one *Plantago Major* plant?
2. Each of the six quadrats measures 1 m by 1 m. For each quadrat, what is the **density** of *Plantago Major* (that is, how many plants/m² are in each of the quadrats)? What is the average density?



Bear Sightings

Goal: Given a set of locations where a bear was recently seen, try to locate the bear.

1. On the map to the right, shade a small circle (the size of a coin) that you think is "close" to all of the points A, B, C, D, E, F, and G.
2. Imagine you had to teach a computer how to choose a region close to those points. How would you describe what to do? Did you add any lines to the map, or compute the distance between pairs of points, or use averages?



3. Suppose that a bear has been sighted at each of the points and a scientist is trying to locate the bear. Where on the map should the scientist focus their search?



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Credits

Slides 9, 10, 16 – 18 and 20 include images from Problems with Purpose, volumes 1 and 2.

Promotional Video

<https://www.youtube.com/watch?v=gOo4QYWiX7Y>

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